MATHEMATICS SOLUTION
(CBCGS SEM – 4 DEC 2019)
BRANCH – IT ENGINEERING

1 a) Find greatest common divisor of the following pairs of integer, using Euclidean algorithm. (3083, 2893). **(05)**

Ans : gcd (3083, 2893)

$3083 = 1\times 2893 + 190$

$2893 = 15\times 190 + 43$

$190 = 4\times 43 + 18$

$43 = 2\times 18 + 7$

$18 = 2\times 7 + 4$

$7 = 1\times 4 + 3$

$4 = 1\times 3 + 1$

$3 = 1\times 3 + 0$

$\therefore \text{gcd}(3083, 2893) = 1$

1 b) Given two lines regression $6y = 5x + 90, 15x = 8y + 130, {\sigma_x}^2 = 16$ **(05)**

Find (i) $\bar{x}$ and $\bar{y}$ (ii)Find r

Ans : 6y = 5x + 90

$\therefore y = \frac{5}{6}x + \frac{90}{6}$

$\therefore y = \frac{5}{6}x + 15 \quad \longrightarrow (1)$

And, $15x = 8y + 13$

$\therefore y = \frac{15}{8}x - \frac{130}{8} \longrightarrow (2)$

Let $b_1 = \frac{5}{6}$ and $b_2 = \frac{15}{8}$

Since $|b_1| < |b_2|$,

$$b_{yx} = b_1 = \frac{5}{6} \ \& \ b_{xy} = \frac{1}{b_2} = \frac{8}{15} \rightarrow (3)$$

$\therefore$ Equation (1) is regression equation of Y and X type and equation (2) is regression equation of X and Y type.

From (1) and (2)

$\frac{5}{6}x + 15 = \frac{15}{8}x - \frac{65}{4}$

$\therefore \frac{65}{4} + 15 = \frac{15}{8}x - \frac{5}{6}x$

$\therefore \frac{125}{4} = \frac{25}{24}x$

$\therefore x = 30$

Substitute x = 30 in (1)

$\therefore y = \frac{5}{6}(30) + 15 = 40$

Now, $r = \pm\sqrt{b_{yx} . b_{xy}}$

$= \pm\sqrt{\frac{5}{6} x \frac{8}{15}}$ (from 3)

$= \pm\frac{2}{3}$

Since, b_{yx} and b_{xy} are both positive, ‘r’ is positive.

$\therefore r = \frac{2}{3} \longrightarrow (4)$

Also, given, ${\sigma_x}^2 = 16$

$\therefore \sigma_x = 4 \longrightarrow (5)$

Using, $b_{yx} = r\frac{\sigma_y}{\sigma_x}$

$\therefore \frac{5}{6} = \frac{2}{3} . \frac{\sigma_y}{\sigma_x}$ (From 3, 4 & 5)

$\therefore \sigma_y = 5$

$\therefore {\sigma_y}^2 = 25$

Ans. 1) $\bar{x} = 30$; $\bar{y} = 40$;

2) $r = \frac{2}{3}$;

3) ${\sigma_y}^2 = 25$

1 c) Prove that A = $\{1,\ 2,\ 3,\ 4,\ 5,\ 6\}$ is a finite abelian group under multiplication modulo. **(05)**

Ans :- We first prepare the table of multiplication modulo 7 denoted by ⊗. From the table it is clear that ⊗ is a binary operation.

⊗	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	4	6	1	3	5
3	3	6	2	5	1	4
4	4	1	5	2	6	3
5	5	3	1	6	4	2
6	6	5	4	3	2	1

G 1: From the table we see that ⊗ is associative.
e.g. 2⊗ (3 ⊗ 5) = 2 ⊗ 1 = 2
and (2 ⊗ 3) ⊗ 5 = 6 ⊗ 5 = 2

G 2 : The first column (or the first row) show that 1 is the identity for ⊗.

G 3 : The positions of the multiplicative identity 1 in every row (and every column) show that every element of A has the multiplicative inverse.
e.g.2 ⊗4 = 1 and 4 ⊗ 2 = 1
∴ $(2)^{-1} = 4$and$(4)^{-1}$ 2
∴ G is a group modulo 7.

G 4 : Further, a ⊗ b = b ⊗ a
e.g. 4 ⊗ 5 = 6 and 5 ⊗ 4 = 6
∴ G is a Abelian Group.

1 d) A random variable x has the following probability function **(05)**

x :	1	2	3	4	5	6	7
P(x) :	K	2K	3K	K^2	K^2+k	$2K^2$	$4K^2$

Find : (i) k (ii) p(x < 5)

Ans :-i) for any probability mass function, $\sum_{i=-\infty}^{+\infty} Pi = 1$

$$\therefore \quad P\,(1) + P(2) + P(3) + P(4) + P(5) + P(6) + P(7) = 1$$

$$\therefore \quad K + 2K + 3K + K^2 + K^2 + K + 2K^2 + 4K^2 = 1$$

$$\therefore 8K^2 + 7K = 1$$

$$\therefore 8K^2 + 7K - 1 = 0$$

$$\therefore K = \frac{1}{8} \qquad or \qquad K = -1 \text{ (Not possible)}$$

$$\therefore K = \frac{1}{8}$$

pmf is ,

∴

X	1	2	3	4	5	6	7
P (x)	$\frac{1}{8}$	$\frac{2}{8}$	$\frac{3}{8}$	$\frac{1}{64}$	$\frac{9}{64}$	$\frac{2}{64}$	$\frac{4}{64}$

ii) P (x< 5)

$$\therefore \quad P\,(x < 5) = p(1) + P(2) + P(3) + P(4)$$

$$= \frac{1}{8} + \frac{2}{8} + \frac{3}{8} + \frac{1}{64}$$

$$= \frac{49}{64}$$

2 a) Calculation coefficient of correlation between x and y **(06)**

x :	**3**	**6**	**4**	**5**	**7**
y :	**2**	**4**	**5**	**3**	**6**

Ans :- n = 5

$$\bar{x} = \frac{\sum X}{n} = \frac{25}{5} = 5$$

$$\bar{y} = \frac{\sum Y}{n} = \frac{20}{5} = 4$$

X	Y	x = X - $\bar{X}$	y = Y - $\bar{Y}$	x^2	y^2	XY
3	2	-2	-2	4	4	4
6	4	1	0	1	0	0
4	5	-1	1	1	1	-1
5	3	0	-1	0	1	0
7	6	2	2	4	4	4
$\sum X = 25$	$\sum Y = 20$			$\sum x^2 = 10$	$\sum y^2 = 10$	$\sum xy = 7$

∴ Coefficient of correlation between x and y,

$$r = \frac{\sum xy}{\sqrt{\sum X^2 . \sum y^2}} = \frac{7}{\sqrt{10.10}} = \frac{7}{10} = 0.7$$

$\therefore \quad r = 0.7$

2 b) A random sample of size 16 from normal population. Showed a mean of 103.75 cm and sum of squares of deviation from the mean 843.75 cm^2 can we say that the population has mean of 108.75 cm ? (06)

n = 16

$\overline{x} = 103.75$

$\in (x - \bar{x})^2 = 83.75$

1) $H_0 : \mu = 108.75$
   $H_1 : \mu \neq 108.75$

2) $t = \frac{\frac{\bar{x} - \mu}{S}}{\sqrt{n-1}}$

$$s = \sqrt{\frac{\in (x - \bar{x})^2}{n}} = \sqrt{\frac{83.75}{16}} = 2.2878$$

$$\therefore \quad t = \frac{\frac{103.75 - 108.75}{2.2878}}{\sqrt{15}} = -8.464$$

$\therefore |t| = 8.464$ (Calculated Value)

3) Table value
Los = 5%
dof = n - 1 = 16-1 = 15

t = 2.131 (table value)

$\therefore$ cal value > table value

$\therefore$ H_0 is rejected

$i.e$ H_1 is accepted

Population does not have a mean of 105.75

2 c) Prove that G = $\{1, -1. i, -i\}$ is a group under usual multiplication of complex numbers. **(08)**

Ans. Let a, b ∈ G
The composition table is

*	1	-1	i	$-i$
1	1	-1	i	$-i$
-1	-1	1	$-i$	i
i	i	$-i$	-1	1
$-i$	$-i$	i	1	-1

From above table, we observe,

a* b exists and a * b∈ G.

∵ *is binary operator in G.

G1:

Multiplication of complex number is associative.

∴ * is associative.

G2:

From table, we observe, first row is same as the header.

∴ 1∈ G is the identity.

∴ Identity exists.

G3:

From table, we observe, identity elements (i.e.1)is present in each row.

∴ $1^{-1} = 1$; $(-1)^{-1} = -1$; $i^{-1} = -i$; $(-i)^{-1} = i$

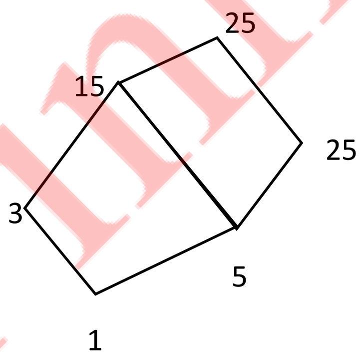
∴inverse of each elements exist and each inverse ∈ G.

∴ Inverse exists. Hence, G is group usual multiplication of complex number.

3 a) Draw Hasse diagram for (D_{75} ≤) , Check whether its a lattice. **(06)**

$Ans : D_{75} = \{ 1, \ 3, \ 5, \ 15, \ 25, \ 75 \}$

Hasse Diagram :



LUB

V	1	3	5	15	25	75
1	1	3	5	15	25	75
3	3	3	15	15	75	75
5	5	15	5	15	25	75
15	15	15	15	15	75	75
25	25	75	25	75	25	75
75	75	75	75	75	75	75

GLB

$\wedge$	1	3	5	15	25	75
1	1	1	1	1	1	1
3	1	3	1	3	1	3
5	1	1	5	5	5	5
15	1	3	5	15	5	15
25	1	1	5	5	25	25
75	1	3	5	15	25	75

Since LUB & GLB exist for all combination D_{75} is lattice.

3 b) Out of 1000 families of 3 children each how many would you expect to have 2 boys and 1 girl? (06)

Ans :-N = 1000

n = 3

p = 0.5

q = 0.5

Using Binomial Distribution, $P\ (x) = \ {}^{n}c_{x} p^{x} q^{n-x}$

$\therefore$ P (2 boys and 1 girl) $= \ {}^{3}C_{2}(0.5)^{2}(0.5)^{1} \times \ {}^{3}C_{1}(0.5)^{1}(0.5)^{2}$

$= (3)(0.125) \times (3)(0.125)$

$= 0.1406$

$\therefore$ Expected Number = Np

= 1000 $\times$ 0.1406

= 140.6

$\approx$ 140

3c) (08)

i) Find last digit of base 7 expansion of 3^{100}i. e 3^{100}(mod 7) by using Fermat's theorem.

Ans : By Fermat's little theorem, $a^{P-1} \equiv 1 \text{ (mod p)}$

$\therefore a = 3 , p = 7$

$3^{7-1} \equiv 1 \text{ (mod 7)}$

$3^{6} \equiv 1 \text{ (mod 7)}$

$3^{96} \equiv 1 \text{ (mod 7)}$

$3^{100} \equiv 81 \pmod 7$

$3^{100} \equiv 4 \pmod 7$

ii) Find the Legendre's symbol $\left(\frac{19}{23}\right)$

$$19 \equiv 3 \ (mod\ 4)$$

$$23 \equiv 3 \ (mod\ 4)$$

$$\therefore \left(\frac{19}{23}\right) = -\left(\frac{23}{19}\right) \rightarrow \quad (1)$$

$$\left(\frac{23}{19}\right) = \left(\frac{4}{19}\right) = \left(\frac{2^2}{19}\right) = 1 \rightarrow \quad (2)$$

∴ from (1) & (2)

$$\left(\frac{19}{23}\right) = -1$$

4 a) Can a complete graph with 8 vertices have 40 edges excluding self – loop. **(06)**

Ans : Complete Graph . (K_n)

A simple graph is complete graph in which every pair of vertices are adjacent.
Degree of every vertex = (n - 1).

No. of edges, $E = \frac{n\,(n-1)}{2}$

∴ for 8 vertices, $E = \frac{8\,(8-1)}{2} = 28$

∴ A complete graph with 8 vertices have 28 edges.

4 b) Find remainder when 2^{50} and 41^{65} are divisible by 7 **(06)**

Ans : (i)We know, $2^3=8=7\times1+1$

∴ $2^3 = 8 \equiv 1 \pmod 7$

∴ $(2^3)^{16} \equiv 1^{16} \pmod 7$

∴ $2^{48} \equiv 1 \pmod 7$

∴ $2^{48} \times 2^2 \equiv 1 \times 2^2 \pmod 7$

∴ $2^{50} \equiv 4 \pmod 7$

Hence, the remainder when 2^{50} is divided by 7 is 4.

(ii) We know , $41 = 5 \times 7 + 6 \therefore \quad 41 \equiv 6(mod\ 7)$

$$41 \equiv 1\ (mod\ 7)$$

$$(41)^{65} \equiv (-1)^{65}\ (mod\ 7)$$

$$41^{65} \equiv -1\ (mod\ 7)$$

$$41^{65} \equiv 6\ (mod\ 7)$$

Hence, the remainder when 41^{65} is divided by 7 is 6.

4 c) Investigate the association between the darkness of eye colour in father and son from the following data: (06)

	Colour of Father's eyes			
Colour of son's eyes		Dark	Not Dark	Total
	Dark	48	90	138
	Not Dark	80	782	862
	Total	128	872	1000

Ans.

Observed Frequency (O)	Expected Frequency (E)	$x^2 = \frac{(o-E)^2}{E}$
48	18	50,0000
90	120	7.5000
80	110	8.1818
782	752	1.1968
	Total	66.8786

Step I:

Null Hypothesis (H_0) : There is no association between the darkness of eye colour in father and son.

Alternative Hypothesis (H_a) : There is association between the darkness of eye colour in father & son. (Two tailed test).

Step 2:

LOS = 5% (Two tailed test)

Degree of Fredom = (r -1) (c-1)

= (2 – 1) (2 – 1)

= 1

∴ Critical value $(x_a{}^2) = 3.841$

Step 3: Test Statistic

$$x_{cal}{}^2 = \sum \frac{(O-E)^2}{E} = 66.8786$$

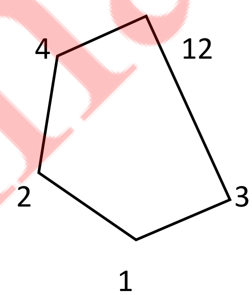
Step 5: Decision

Since $x_{cal}{}^2 > x_a{}^2$, H_0 is rejected.

∴ There is association between the darkness of eye colour in father and son.

5 a) Let L = { 1, 2, 3, 4, 12 } and the relation be " is divisible by " write compliments of L . **(06)**

Ans : Hasse Diagram



Elements	1	2	3	4	12
Compliments	12	3	2,4	3	1

5 b) If x is a Poisson variate and p (x = 0)= 6p (x= 3) Find P (x = 2) **(06)**

Ans : Using Poisson Distribution, $P(x) = \frac{e^{-m}.m^x}{x!}$

P(x = 0) = 6 P (x = 3)

$$\frac{e^{-m}.m^0}{0!} = 6 \quad \frac{e^{-m}.m^3}{3!}$$

$$\therefore \; 1 = m^3$$

Or $m = 1$

$$\therefore \; P\,(x = 2) = \frac{e^{-1}.\,1^2}{2!} = 0.1839$$

5 c) Define the following terms giving illustration. **(08)**

1.	**Simple graph**	**2.**	**Complete graph**
3.	**Bipartite graph**	**4.**	**Planar graph**

1. <u>**Simple Graphs (K)**</u>

→ A graph in which there are No loops, No Multiple edges and every vertex has same degree.

No. of Edges, $E = \frac{n \times d}{2}$, $\quad n \rightarrow$ *no.of vertex .* $\quad d \rightarrow$ *degree of vertex .*

Example :-

$$\therefore \; E = \frac{4 \times 2}{2} = 4$$

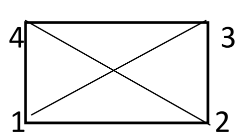
2. <u>**Complete Graph . (K_n)**</u>

→ A simple graph is complete graph in which every pair of vertices are adjacent.

Degree of every vertex = (n - 1). No. of edges, $E = \frac{n\,(n-1)}{2}$

Example :-

<u>4 Vertices</u>



$$\therefore \; E = \frac{4\,(4-1)}{2} = 6$$

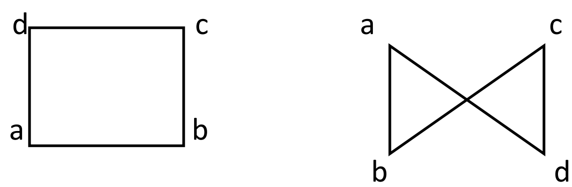
3. Bipartite Graph

→ A graph G (V, E) is called Bipartite (= of two parts) if

(i) V can be expressed as a Union of two disjoint sets U and W
(V = U∪W and U∩W= ∅)

(ii) Every edge in E has one vertex in U and the Other in W.

Example



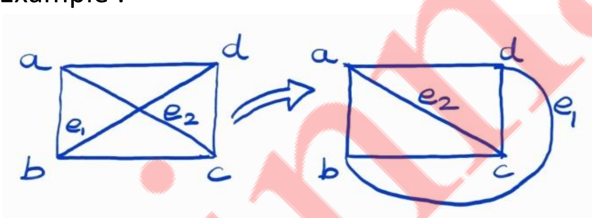
$$\therefore \quad U = \{a, c\}, \quad W = \{b, d\}$$

$$\therefore \ V = U \cup W \ \& \ U \cap W = \emptyset$$

4. Planar Graph

→ The Graph drawn in such a way that no two edges intersect, is called a Planar Graph.

Example :-



6 a) Solve : x =1 (mod 5) , x= 2 (mod 6) , x=3 (mod 7) (06)

Ans:

$a_1 = 1$ $m_1 = 5$

$a_2 = 2$ $m_2 = 6$

$a_3 = 3$ $m_3 = 7$

$M = m_1.m_2.m_3$

$M = 5 \times 6 \times 7 = 210$

$M_1 = 6 \times 7 = 42$

$M_2 = 5 \times 7 = 35$

$M_3 = 5 \times 6 = 30$

$M_1 x \equiv 1(mod\ m_1)$
$42x \equiv 1(mod\ 5)$
$1 \equiv 42x\ (mod\ 5)$
$1 \equiv 2x(mod\ 5)$
$3 \equiv 6x(mod\ 5)$
$3 \equiv x\ (mod\ 5)$
$x \equiv 3\ (mod\ 5)$

$M_2 x \equiv 1(mod\ m_2)$
$35x \equiv 1(mod\ 6)$
$1 \equiv 35x\ (mod\ 6)$
$1 \equiv 5x\ (mod\ 6)$
$1 \equiv -x\ (mod\ 6)$
$x \equiv\ -1\ (mod\ 6)$
$x \equiv 5\ (mod\ 6)$

$M_3 x \equiv 1(mod\ m_3)$
$30x \equiv 1(mod\ 7)$
$1 \equiv 30x(mod\ 7)$
$1 \equiv 2x\ (mod\ 7)$
$4 \equiv 8x\ (mod\ 7)$
$4 \equiv x\ (mod\ 7)$
$x \equiv 4\ (mod\ 7)$

By Chinese Remainder Theorem,

$\therefore \quad x \equiv [a_1 M_1 x_1 + a_2 M_2 x_2 + a_3 M_3 x_3]\ modulo\ \ M.$

$x \equiv [(1)(42)(3) + (2)(35)(5) + (3)(30)(4)]\ modulo\ 210$

$x \equiv 836\ (mod\ 210)$

$x \equiv 206\ (mod\ 210)$

$\therefore$ $\boxed{x = 206}$ is one solution . General solution is given by, $\boxed{x = 206 + 210\ k}$ where k is any integer

6 b) A certain injection administered to 12 patients resulted in following changes of blood pressure (5, 2, 8, -1, 3, 0, 6, -2, 1, 5, 0, 4) can it be concluded that injection will be in general accompanied by an increasein blood pressure ? (06)

Ans : n = 12

H_0 : ? = 0
H_1 : ? ≠ 0

X	$d\ =\ x - a$	d^2
5	3	9
2	0	0
3	6	36
-1	-3	9
3	1	1
0	-2	4
6	4	16
-2	-4	16

1	-1	1
5	-3	4
0	-2	4
4	2	4

$Let \quad a = 2$

$\in d = 7 \qquad \in d^2 = 109$

$$\bar{x} = a + \frac{\in d}{n}$$

$$\therefore \quad \bar{x} = \frac{2+7}{12} = 2.5833$$

$$\in (x-\bar{x})^2 = \in d^2 - \frac{(\in d)^2}{n}$$

$$= 109 - \frac{(7)^2}{12}$$

$$= 104.916$$

$$s = \sqrt{\frac{\in (x-\bar{x})^2}{n}} = \sqrt{\frac{104.916}{12}} = 2.956$$

$$t = \frac{\bar{x}}{\frac{s}{\sqrt{n-1}}} = \frac{2.5833}{\frac{2.956}{\sqrt{II}}} = 2.898 \quad \text{(Calculated Value)}$$

Table value

Los → 5%

dof→ n - 1 = 11

$\therefore \quad k = 2.201$ (Table value)

Cal cal > Table value

$\therefore \quad H_0$ is rejected , H_1 is accepted.

$\therefore$ It can be concluded that injection will in general accompanied by increase B. P.

6 c) **(08)**

(i)Write the following permutation as product of disjoint cycles.

f = (1 3 2 5) (1 4 5) (2 5 1)

Ans.

$f(1) = (1\ 3\ 2\ 5)(1\ 4\ 5)\ (2\ 5\ 1)\ \ (1)$

$= (1\ 3\ 2\ 5)\ (1\ 4\ 5\ )\ (2)$

$= (1\ 3\ 2\ 5)\ (2)$

$\therefore f(1) = 5$

$f(2) = (1\ 3\ 2\ 5)(1\ 4\ 5)\ (2\ 5\ 1)\ \ (2)$

$= (1\ 3\ 2\ 5)\ (1\ 4\ 5)\ (5)$

$= (1\ 3\ 2\ 5)\ (1)$

$\therefore f(2) = 3$

$f(3) = (1\ 3\ 2\ 5)\ (1\ 4\ 5)\ (2\ 5\ 1)\ \ (3)$

$= (1\ 3\ 2\ 5)\ (1\ 4\ 5)\ (3)$

$= (1\ 3\ 2\ 5)\ (3)$

$\therefore f(3) = 2$

$f(4) = (1\ 3\ 2\ 5)\ (1\ 4\ 5)\ (2\ 5\ 1)\ \ (4)$

$= (1\ 3\ 2\ 5)\ (1\ 4\ 5)\ (4)$

$= (1\ 3\ 2\ 5\ )\ (5)$

$\therefore f(4) = 1$

$f(5) = (1\ 3\ 2\ 5)\ (1\ 4\ 5)\ (2\ 5\ 1)\ \ (5)$

$= (1\ 3\ 2\ 5)\ (1\ 4\ 5)\ (1)$

$= (1\ 3\ 2\ 5)(4)$

$\therefore f(5) = 4$

$\therefore f = \begin{pmatrix} 1\ 2\ 3\ 4\ 5 \\ 5\ 3\ 2\ 1\ 4 \end{pmatrix}$

Hence, expressing permutation f as the product of disjoint cycle we have

$f = (1\ 5\ 4)\ (2\ 3\ )$

(ii) Simplify as sum of product (A+B) (A +B')(A'+B)(A'+B')

Ans. Consider,

(A+B) (A +B') (A'+B)(A'+B')

≡[(A +B)(A'+B')][(A + B') (A'+ B)]

≡[A (A' +B')+B(A'+B')][A (A'+ B)+B'(A'+ B)]

≡[(AA'+AB'+BA'+BB'][AA'+AB+B'A+B'B]

≡ [0+AB'+BA'+0][0+AB+B'A'+0] (Complement Law)

≡ [AB'+BA'][AB+B'A'] (Identify law)

≡ AB' [AB +B'A'] + BA' [AB +B'A']

≡ (AB')(AB) + (AB')(B'A')+(BA')(AB)+(BA')(B'A')

≡ (AB')(BA)+(B'A)(A'B')+(BA')(AB)+(A'B)(B'A') (Complement Law)

≡ A (B'B)A+B'(AA')B'+B(A'A)B+A'(BB')A' (Associative Law)

≡A(0)A+B'(0)B'+B(0)B+A'(0)A' (Complement Law)

≡ 0 + 0+0+0 (Domination Law)

≡ 0 (idempotent Law)

∴ (A +B)(A+B')(A'+B)(A'+B') = 0